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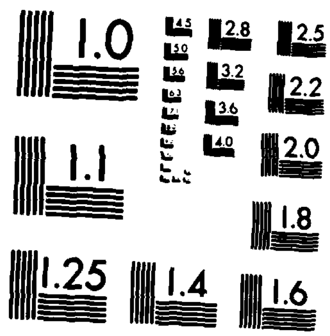

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FIXED WIDTH INTERVAL ESTIMATION IN LINEAR REGRESSION

BY

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1. INTRODUCTION

Stein (1945) describes a two-stage procedure to obtain a fixed-width confidence interval for the mean of a normal population when the variance is unknown. This is followed by works of Anscombe (1953) and Chow and Robbins (1965) who advocate sequential procedures. Hall (1981) suggests a three-stage sampling technique that combines the simplicity of Stein's procedure with the efficiency of the fully sequential method. For a linear model $Y_i = X_i\beta + \varepsilon_i$ where ε_i is $N(0,\sigma^2)$, the corresponding problem of obtaining a fixed-width confidence interval for one of the parameters is more difficult since the variance of the usual estimate depends not only on σ^2 but also on the X_i. To avoid this difficulty, Stein (1945) assumes that $X_1, \ldots, X_m$ are fixed and that they are repeated as a whole, as many times as is necessary. For example, $X_1, \ldots, X_m$ may correspond to an orthogonal design which we are replicating. Bishop (1978) continues to assume that the X_i are fixed.

In this paper, we consider simple linear regression $Y_i = \gamma + \beta X_i + \varepsilon_i$ where ε_i is $N(0,\sigma^2)$ and X_i is $N(\mu,\tau^2)$. In other words, we are sampling from a bivariate normal population. In section 2, we describe a two-stage procedure to obtain a fixed-width confidence interval for β and prove that the specified coverage probability is attained. Essentially, we estimate σ^2 and predict X_n, $n > m$ on the basis of a pilot sample $(X_1,Y_1), \ldots, (X_m,Y_m)$ to determine the size of the second sample. If we sample sequentially, then there is no need to predict X_n, $n > m$; such a procedure is described in section 3. We show that the corresponding confidence interval attains the specified coverage probability regardless of the distribution of the X_i. The procedure behaves like Stein's procedure for the estimation of a normal mean. By updating the estimate of σ^2 sequentially, we arrive at another procedure. Section 4 deals with the related problem of deriving a test procedure of $H : \beta = \beta_0$ at level α_0 which has power at least α_1 at $\beta = \beta_0 + \Delta$ independent of the values of the other parameters. One way to construct such a test makes use of fixed-width confidence

intervals for β. A different approach which treats X_i and Y_i symmetrically is based on the distribution of the sample correlation coefficient. We show that the resulting test attains the specified level and power asymptotically.

2. A TWO-STAGE PROCEDURE

Suppose that σ^2 is known and the X_i are known constants, then $\hat\beta_n$ is $N(\beta,\sigma^2/\sum_1^n(X_i-\bar X_n)^2)$ where $\hat\beta_n=\sum_1^n(X_i-\bar X_n)Y_i/\sum_1^n(X_i-\bar X_n)^2$ is the least squares estimate of β based on $(X_1,Y_1), \ldots, (X_n,Y_n)$. It follows that $P(|\hat\beta_n-\beta|<d)\geq 1-\alpha$ if

$$\sum_1^n(X_i-\bar X_n)^2 \geq Z^2_{1-\alpha/2}\sigma^2/d^2 = S_0$$

where $Z_{1-\alpha/2}$ stands for the $(1-\alpha/2)$-percentile of the standard normal distribution. Since σ^2 is unknown and the X_i are stochastic, we need to estimate σ^2 and predict X_n, $n>m$ on the basis of the pilot sample $(X_1,Y_1), \ldots, (X_m,Y_m)$, $m\geq 3$. An obvious estimate of σ^2 is $\hat\sigma^2_m=\sum_1^m(Y_i-\hat\gamma_m-\hat\beta_m X_i)^2/(m-2)$. To reduce the prediction problem, we note that we only need to predict $\sum_1^n(X_i-\bar X_n)^2$ for $n>m$. Since X_i is $N(\mu,\tau^2)$, we make the Helmert transformation to obtain $\sum_1^m(X_i-\bar X_m)^2=\tau^2(U_2^2+\ldots+U_m^2)$ and $\sum_1^n(X_i-\bar X_n)^2=\tau^2(U_2^2+\ldots+U_m^2+\ldots+U_n^2)$ for $n>m$ where U_2, U_3, ... are independent standard normal variables. This allows us to make use of standard results of prediction for the gamma case. In particular, if $b_n=1+\chi^2_{1-c}(n-m)/\chi^2_g(m)$ where $\chi^2_{1-c}(n-m)$ and $\chi^2_g(m)$ are chi-square percentiles, then for each $n>m$, $(b_n\sum_1^m(X_i-\bar X_m)^2,\infty)$ is a (c,g) guaranteed coverage interval predictor of $\sum_1^n(X_i-\bar X_n)^2$ (Aitchison & Dunsmore 1975, Ch.6). Furthermore, we can guarantee coverage simultaneously so that with probability g, the pilot sample $X_1, \ldots, X_m$ is such that $P(\sum_1^n(X_i-\bar X_n)^2>b_n\sum_1^m(X_i-\bar X_m)^2|X_1,\ldots,X_m)\geq c$ for each $n>m$. We choose c, g so that $cg>1-\alpha$ and define α' by $1-\alpha=cg(1-\alpha')$. For conveience, we let $b_m=1$. Consider the following two-stage sampling procedure.

Procedure 1. (i) Obtain a pilot sample $(X_1,Y_1), \ldots, (X_m,Y_m)$ and calculate $\hat\gamma_m$, $\hat\beta_m$ and $\hat\sigma^2_m$.

(ii) Draw a second sample of size $N_1 - m$ where N_1 is the smallest $n \geq m$ such that $b_n \sum_1^m (X_i - \bar{X}_m)^2 \geq t^2_{1-\alpha'/2}[m-2]\hat{\sigma}^2_m/d^2$ and $t_{1-\alpha'/2}[m-2]$ is the $(1-\alpha'/2)$-percentile of a t distribution with m-2 degrees of freedom.

The following theorem says that $(\hat{\beta}_{N_1} - d, \hat{\beta}_{N_1} + d)$ is a $(1-\alpha)$-level confidence interval for β.

Theorem 1. $P(|\hat{\beta}_{N_1} - \beta| < d) \geq 1 - \alpha$.

Before we prove theorem 1, we first state two lemmas.

Lemma 1. The conditional distribution of $\hat{\beta}_{N_1}$ *given* $\hat{\sigma}_m$ *and* $X_1, X_2, \ldots$ *is* $N(\beta, \sigma^2/S_1)$ *where* $S_1 = \sum_1^{N_1}(X_i - \bar{X}_{N_1})^2$.

Proof. Given $X_1, X_2, \ldots, N_1$ depends only on $\hat{\sigma}^2_m$ and $\hat{\beta}_{N_1}$ can be written as a linear combination of $\bar{Y}_m$, $\hat{\beta}_m$ and $Y_{m+1}, \ldots, Y_{N_1}$, all of which are independent of $\hat{\sigma}^2_m$.

Lemma 2. $P(\sum_1^{N_1}(X_i - \bar{X}_{N_1})^2 > b_{N_1}\sum_1^m(X_i - \bar{X}_m)^2 | \hat{\sigma}_m) \geq gc$.

Proof. Let $A = \{(x_1, \ldots, x_m) : \forall\, n > m,\ P(\sum_1^n (X_i - \bar{X}_m)^2 > b_n \sum_1^m (X_i - \bar{X}_m)^2 | X_1 = x_1, \ldots, X_m = x_m) \geq c\}$, then $P((X_1, \ldots, X_m) \in A) = g$ by our choice of b_n. Since $\hat{\sigma}_m$ is independent of the X_i, we also have $P((X_1, \ldots, X_m) \in A | \hat{\sigma}_m) = g$. If $(X_1, \ldots, X_m) = (x_1, \ldots, x_m) \in A$ and we write $n_1 = N_1(\hat{\sigma}_m, x_1, \ldots, x_m)$, then

$$
\begin{aligned}
&P(\sum_1^{N_1}(X_i - \bar{X}_{N_1})^2 > b_{N_1}\sum_1^m(X_i - \bar{X}_m)^2 | \hat{\sigma}_m, X_1 = x_1, \ldots, X_m = x_m) \\
&= P(\sum_1^{n_1}(X_i - \bar{X}_{N_1})^2 > b_{n_1}\sum_1^m(X_i - \bar{X}_m)^2 | \hat{\sigma}_m, X_1 = x_1, \ldots, X_m = x_m) \\
&= P(\sum_1^{n_1}(X_i - \bar{X}_{N_1})^2 > b_{n_1}\sum_1^m(X_i - \bar{X}_m)^2 | X_1 = x_1, \ldots, X_m = x_m) \\
&\geq c.
\end{aligned}
$$

Combining, we have the desired result.

Corollary 1. $P(\sum_1^{N_1}(X_i - \bar{X}_{N_1})^2 > t^2_{1-\alpha'/2}[m-2]\hat{\sigma}^2_m/d^2 | \hat{\sigma}_m) \geq gc$.

Proof. This follows from lemma 2 and the definition of N_1.

We now prove theorem 1.

$$P(|\hat{\beta}_{N_1} - \beta| < d | \hat{\sigma}_m)$$

$$= E_{X_1, X_2 \ldots | \hat{\sigma}_m} \{P(|\hat{\beta}_{N_1} - \beta| < d | \hat{\sigma}_m, X_1, X_2, \ldots)\}$$

$$= E\{2\Phi(d\sqrt{S_1}/\sigma) - 1 | \hat{\sigma}_m\} \qquad \text{by lemma 1}$$

$$\geq gc\{2\Phi(t_{1-\alpha'/2}[m-2]\hat{\sigma}_m/\sigma) - 1\} \qquad \text{by corollary 1.}$$

Thus $P(|\hat{\beta}_{N_1} - \beta| < d) \geq gc\ E\{2\Phi(t_{1-\alpha'/2}[m-2]\hat{\sigma}_m/\sigma) - 1\}$

$$= gc(1 - \alpha')$$

$$= 1 - \alpha.$$

3. SEQUENTIAL PROCEDURES

If we sample sequentially, then prediction is no longer necessary.

Procedure 2. (i) Obtain a pilot sample of size m. (ii) Sample sequentially until $\sum_1^n (X_i - \bar{X}_n)^2 \geq t^2_{1-\alpha/2}[m-2]\hat{\sigma}^2_m/d^2$.

Let N_2 be the sample size when we terminate sampling, our next theorem asserts that $(\hat{\beta}_{N_2} - d, \hat{\beta}_{N_2} + d)$ is a $(1-\alpha)$-level confidence interval for β.

Theorem 2. $P(|\hat{\beta}_{N_2} - \beta| < d) \geq 1 - \alpha$.

We first state a lemma.

Lemma 3. The conditional distribution of $\hat{\beta}_{N_2}$ *given* $\hat{\sigma}_m$ *and* X_1, X_2, ... *is* $N(\beta, \sigma^2/S_2)$ where $S_2 = \sum_1^{N_2} (X_i - \bar{X}_{N_2})^2$.

This is the analog of lemma 1 and can be proved using similar technique. We now prove theorem 2.

$$P(|\hat{\beta}_{N_2} - \beta| < d) = E\{P(|\hat{\beta}_{N_2} - \beta| < d | \hat{\sigma}_m, X_1, X_2, \ldots)\}$$

$$= E\{2\Phi(d\sqrt{S_2}/\sigma) - 1\} \qquad \text{by lemma 3}$$

$$\geq E\{2\Phi(t_{1-\alpha/2}[m-2]\hat{\sigma}_m/\sigma) - 1\}$$

$$= 1 - \alpha.$$

We note that theorem 2 holds even when the X_i are not normally

distributed.

Since the estimate of σ^2 is not updated as we sample sequentially, procedure 2 is inefficient. It behaves like Stein's procedure for the estimation of the mean of a normal population. In fact

$$E(S_2) = E(\sum_1^{N_2}(X_i - \bar{X}_{N_2})^2)$$

$$\geq E(t^2_{1-\alpha/2}[m-2]\hat{\sigma}^2_m/d^2)$$

$$= S_0 t^2_{1-\alpha/2}[m-2]/Z^2_{1-\alpha/2}$$

so that $E(S_2)/S_0 = t^2_{1-\alpha/2}[m-2]/Z^2_{1-\alpha/2} > 1$.

If the estimate of σ^2 is updated sequentially, we obtain the following procedure.

Procedure 3. (i) Obtain a pilot sample of size m. (ii) Sample sequentially until $\sum_1^n(X_i - \bar{X}_n)^2 \geq a_n^2\hat{\sigma}_n^2/d^2$ where $\{a_n\}$ is a sequence of constants converging to $Z_{1-\alpha/2}$.

We expect procedure 3 to be the most efficient, but unlike procedures 2 and 3, the specified coverage probability is attained only asymptotically. Procedure 1 is least efficient since we have to deal with the additional problem of prediction, however, it has the advantage of requiring only two sampling operations.

4. A RELATED PROBLEM

A problem related to fixed-width interval estimation of β is that of deriving a test procedure of $H : \beta = \beta_0$ at level α_0 which has power at least α_1 at $\beta = \beta_0 + \Delta$, $\Delta > 0$. We can make use of our earlier results to solve this problem. For instance, we can use procedure 2 to obtain a $(1-\alpha)$-level confidence interval for β with width 2d, $d < \Delta$ and reject H if β_0 lies outside that interval. The resulting test has level α_0 and its power at $\beta = \beta_0 + \Delta$ is

$$P_{\beta_0+\Delta}(|\hat{\beta}_{N_2} - \beta_0| > d) \geq P_{\beta_0+\Delta}(\hat{\beta}_{N_2} > \beta_0 + d)$$

$$= E\{P_{\beta_0+\Delta}(\hat{\beta}_{N_2} > \beta_0 + d|\hat{\sigma}_m, X_1, \ldots)\}$$

$$= E\{1 - \Phi((d-\Delta)\sqrt{S_2}/\sigma)\}$$

$$\geq E\{1 - \Phi((d-\Delta)t_{1-\alpha_0/2}[m-2]\hat{\sigma}_m/d\sigma)\}.$$

If we choose d such that $(\Delta - d)t_{1-\alpha_0/2}[m-2]/d = t_{\alpha_1}[m-2]$, then the power is at least α_1. As expected, if $d = \Delta$, then the power is at least $\frac{1}{2}$; as $d \to 0$, the power increases to 1.

The technique we employ so far is to condition on the X_i and then treat them as if they are fixed. An unconditional approach treating the X_i and Y_i symmetrically is described below. Without loss of generality, the hypothesis is $H : \beta = 0$. Assume that we are sampling from a bivariate normal population

$$\binom{X_i}{Y_i} \sim N_2\left(\binom{\mu}{\xi}\begin{pmatrix}\tau^2 & \rho\tau\nu\\ \rho\tau\nu & \nu^2\end{pmatrix}\right)$$

then H is equivalent to $\rho = 0$ and the usual t test rejects H if $|r|$ is too large where r is the sample correlation coefficient. Since the distribution of r depends on the parameters only through ρ, we can determine the sample size such that the level - α test of $\rho = 0$ has power α_1 at another ρ value. Bock (1977) makes use of Fisher Z-transformation to derive an approximate formula for the required sample size

$$Z_{1-\alpha_0/2} - (n-3)^{\frac{1}{2}}\tanh^{-1}\rho = Z_{1-\alpha_1}.$$

Since $\rho = \theta/(1+\theta^2)^{\frac{1}{2}}$ where $\theta = \beta\tau/\sigma$, the following procedure suggests itself.

Procedure 4. (i) Obtain a pilot sample of size m. (ii) Sample sequentially until $Z_{1-\alpha_0/2} - (n-3)^{\frac{1}{2}}\tanh^{-1}\hat{\rho}_n(\Delta) \leq Z_{1-\alpha_1}$ where $\hat{\rho}_n(\Delta) = \hat{\theta}_n(\Delta)/(1+\hat{\theta}_n(\Delta))^{\frac{1}{2}}$, $\hat{\theta}_n(\Delta) = \Delta\hat{\tau}_n/\hat{\sigma}_n$ and $\hat{\tau}_n^2 = \sum_1^n (X_i - \bar{X}_n)^2/(n-1)$. (iii) Perform a two-sided t test treating the final sample size $N(\Delta)$ as if it is fixed. Thus if $T_n = \hat{\beta}_n(\sum_1^n (X_i - \bar{X}_n)^2)^{\frac{1}{2}}/\hat{\sigma}_n$, we reject H if $|T_{N(\Delta)}| > t_{1-\alpha_0/2}[N(\Delta)-2]$.

The following theorem asserts that the test procedure attains the specified level and power asymptotically.

Theorem 3. $\lim_{\Delta\to 0} P_{\beta=0}(|T_{N(\Delta)}| < t_{1-\alpha_0/2}[N(\Delta)-2]) = 1-\alpha_0,$

$$\lim_{\Delta\to 0} P_{\beta=\Delta}(|T_{N(\Delta)}| > t_{1-\alpha_0/2}[N(\Delta)-2]) \geq \alpha_1.$$

Proof. (i) Since $r_n = T_n/(n-2+T_n^2)^{\frac{1}{2}}$ where r_n is the sample correlation coefficient computed from $(X_1,Y_1), \ldots, (X_n,Y_n)$,

$$\begin{aligned} 1-\alpha_0 &= P_{\beta=0}(|T_n| < t_{1-\alpha_0/2}[n-2]) \\ &= P_{\beta=0}(|(n-3)^{\frac{1}{2}}\tanh^{-1} r_n| < C_n) \end{aligned}$$

where $C_n = (n-3)^{\frac{1}{2}}\tanh^{-1}(t_{1-\alpha_0/2}[n-2]/(n-2+t^2_{1-\alpha_0/2}[n-2])^{\frac{1}{2}})$. On the other hand, when $\beta = 0$

$$(n-3)^{\frac{1}{2}}\tanh^{-1} r_n \to_{\mathcal{D}} N(0,1) \text{ as } n \to \infty,$$

so we must have $\lim_{n\to\infty} C_n = Z_{1-\alpha_0/2}$. Since $N(\Delta) \to \infty$ a.s. as $\Delta \to 0$, $\lim_{\Delta\to 0} C_{N(\Delta)} = Z_{1-\alpha_0/2}$ a.s. and it follows from a theorem of Anscombe (1952) that when $\beta = 0$

$$(N(\Delta)-3)^{\frac{1}{2}}\tanh^{-1} r_{N(\Delta)} \to_{\mathcal{D}} N(0,1).$$

Thus $\lim_{\Delta\to 0} P_{\beta=0}(|T_{N(\Delta)}| < t_{1-\alpha_0/2}[N(\Delta)-2])$

$$\begin{aligned} &= \lim_{\Delta\to 0} P_{\beta=0}(|(N(\Delta)-3)^{\frac{1}{2}}\tanh^{-1} r_{N(\Delta)}| < C_{N(\Delta)}) \\ &= 1-\alpha_0. \end{aligned}$$

(ii) Assume for the time being that under $\beta = \Delta$

$$(N(\Delta)-3)^{\frac{1}{2}}\tanh^{-1} r_{N(\Delta)} \to_{\mathcal{D}} N(Z_{1-\alpha_0/2} - Z_{1-\alpha_1}, 1) \text{ as } \Delta \to 0, \quad (1)$$

then $\lim_{\Delta\to 0} P_{\beta=\Delta}(|T_{N(\Delta)}| > t_{1-\alpha_0/2}[N(\Delta)-2])$

$$\begin{aligned} &\geq \lim_{\Delta\to 0} P_{\beta=\Delta}((N(\Delta)-3)^{\frac{1}{2}}\tanh^{-1} r_{N(\Delta)} > C_{N(\Delta)}) \\ &= \alpha_1. \end{aligned}$$

To prove (1), we fix γ, σ, μ, τ and define $n(\Delta) - 3$ to be the least integer greater than or equal to $(Z_{1-\alpha_0/2} - Z_{1-\alpha_1})^2/(\tanh^{-1}\rho(\Delta))^2$ where $\rho(\Delta) = \theta(\Delta)/(1+\theta(\Delta))^{\frac{1}{2}}$ and $\theta(\Delta) = \Delta\tau/\sigma$. Under $\beta = \Delta$

$$(n(\Delta)-3)^{\frac{1}{2}}(\tanh^{-1} r_{n(\Delta)} - \tanh^{-1}\rho(\Delta)) \to_{\mathcal{D}} N(0,1) \quad \text{as} \quad \Delta \to 0,$$

equivalently

$$(n(\Delta)-3)^{\frac{1}{2}}\tanh^{-1} r_{n(\Delta)} \xrightarrow{\mathcal{D}} N(Z_{1-\alpha_0/2} - Z_{1-\alpha_1}, 1) \quad \text{as} \quad \Delta \to 0 \qquad (2)$$

from which (1) follows if we can replace $n(\Delta)$ by $N(\Delta)$. To that end, we note that if X_i is $N(\mu,\tau^2)$ and Y_i is $N(\gamma,\sigma^2)$ independently of X_i, then the conditional distribution of $Y_i + \beta X_i$ given X_i is $N(\gamma + \beta X_i, \sigma^2)$. The advantage of this representation is that it enables us to deal with a single array of random variables rather than a double array. In particular, we can show $N(\Delta)/n(\Delta) \to 1$ a.s. as $\Delta \to 0$. A generalization of Anscombe's theorem enables us to replace $n(\Delta)$ by $N(\Delta)$ in (2), we omit the details.

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We discuss fixed-width interval estimation for the slope parameter β in a simple linear regression $Y_i = \gamma + \beta X_i + \varepsilon_i$ when the X_i are also normally distributed. A two-stage procedure that combines prediction with estimation is described. In addition, we discuss two sequential procedures. The confidence intervals obtained are used to construct tests of $H: \beta = \beta_o$ with level α_0 and power at least α_1 at $\beta = \beta_o + \Delta$ independent of the values of the other parameters. We also consider a sequential procedure based on the distribution of the sample correlation coefficient; the resulting test attains the specified level and power asymptotically.

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